Error Rate for PSK Modulation over AWGN Channel

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***Abstract*—This report presents an analysis of phase-shift keying modulation over an additive white gaussian noise channel. The communication system comprises of a pseudo-random bit generator, PSK modulation, transmission over an AWGN Channel, and an optimal detector at the receiver. The analysis was performed with python software to simulate the described communication system for different values of noise variance for the channel. The results obtained by the simulation describe the bit error rate and symbol error rate for PSK modulation as a function of signal-to-noise ratio. This study compares error rates for BPSK, QPSK, and 8-PSK and further investigates improvements that can be achieved when implementing the Viterbi algorithm and convolution encoding.**

I. INTRODUCTION

The M-PSK digital modulation technique is utilized in various applications including satellite communication, wireless communication, and digital broadcasting [1]. The M-PSK modulation scheme involves mapping bits to symbols that represent a particular phase for a signal. When this signal is transmitted across the channel, an optimal detector at the receiver decodes the signal to get the original bits that were mapped to that symbol. When the transmitted signal is subjected to noise, the phase may be altered, so the optimal detector utilizes decision boundaries to detect symbols. If the channel noise is large enough, a received signal could be detected incorrectly due to the phase crossing the decision boundary. This consequently results in bit errors ultimately presenting a loss of data integrity after communication.

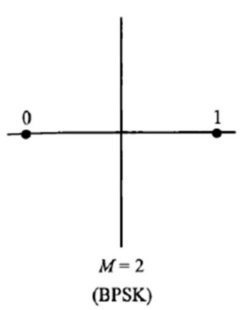

Fig. 1. BPSK bit to symbol mapping

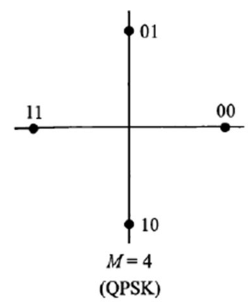

Fig. 2. QPSK bit to symbol mapping

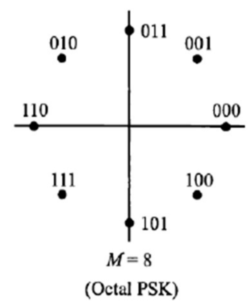

Fig. 3. 8-PSK bit to symbol mapping

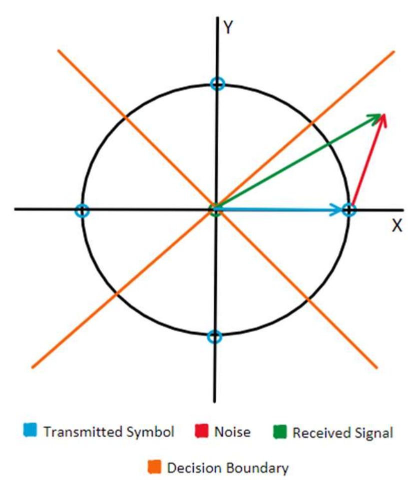

Fig. 4. Illustration of channel noise effect on PSK modulation

This study aims to find the error rate of the optimal detector as a function of signal-to-noise ratio (SNR). To find results for symbol error rate (SER) and bit error rate (BER) as a function of

SNR, a simulation was performed with adjusted values of variance for the x and y-components of the noise signal vector. The simulation was performed for BPSK, QPSK, and 8-PSK modulation.

BPSK is a digital modulation that involves mapping one bit to one symbol. Because there are two symbols for BPSK, the phase angles of each neighboring symbol are offset by 180°. This means that decision boundaries for phase detection are set ±90° away from each symbol.

QPSK is a digital modulation that provides a larger spectral efficiency than BPSK. This is due to the larger number of symbols used for bit mapping. In this case, two bits are mapped to one symbol, delivering twice the amount of data per transmitted signal when compared with BPSK. The phase angles of each neighboring symbol are offset by 90° for QPSK. The decision boundaries for phase detection are set ±45° away from each symbol. While the spectral efficiency of QPSK is larger, this smaller region between decision boundaries results in a larger SER at the detector. This is a tradeoff that must be considered when designing a communication system with certain performance specifications.

8-PSK provides an even larger spectral efficiency over BPSK. Three bits are mapped to one symbol, delivering triple the amount of data per transmitted signal when compared with BPSK. The phase angles of each neighboring symbol are offset by 45° for 8-PSK. The decision boundaries for phase detection are set ±22.5° away from each symbol. 8-PSK demonstrates another tradeoff between spectral efficiency and error rate at the detector.

The communication system used for the simulation comprised of several elements that can be simplified to a block diagram. The first block is a pseudo random bit generator used to generate the message that will be modulated for transmission. This message is passed to the modulator where the message will either be modulated using BPSK, QPSK, or 8-PSK. This modulated signal is then transmitted over an AWGN channel with parameter σ^2 representing the variance of noise. The receiver then uses an optimal detector to recover symbols and decode back to bits.

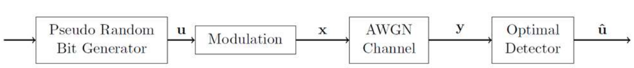

Fig. 5. Block diagram of the communication system

The second part of the simulation implements convolutional encoding and the Viterbi algorithm for error correction to show performance improvements that can be achieved for BER of PSK modulation. Convolutional encoding introduces redundancy in the transmitted signal to improve its resilience to errors caused by noise in the channel. The Viterbi algorithm uses a trellis structure to find the most likely sequence of transmitted bits given the received signal. While the SER of the channel remains the same as a function of SNR, the Viterbi algorithm provides improvement for bit error correction at the receiver.

II. Implementation of Simulation Software

The software used to simulate the M-PSK communication system was implemented with python. The python software uses a terminal interface that allows a user to select the modulation type between BPSK, QPSK, and 8-PSK. The program also prompts the user for a noise variance value of the AWGN Channel. SER and BER outputs from the simulation were written to a CSV file so that the data could be stored for further analysis.

The message sample size used for this simulation was six hundred thousand bits. The transmitted symbols had an energy ϵ_s normalized to one to provide a reference point with respect to channel noise. Noise energy N_0 was calculated as a function of the noise variance σ^2 so that the simulation outputs could be represented in terms of SNR.

$$N_0 = 2\sigma^2$$

$$SNR = \frac{\epsilon_s}{N_0}$$

The second part of the simulation incorporates an option to perform error correction on transmission using convolutional encoding and the Viterbi algorithm. This part of the simulation used a message sample size of one hundred twenty thousand bits. The code was generated by a (2,1,3) convolutional encoder providing a code rate of $1/2$.

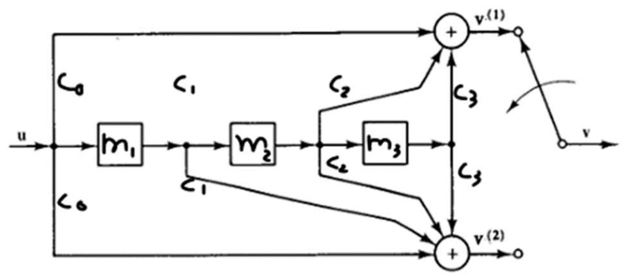

Fig. 6. (2,1,3) binary convolutional encoder

TABLE I
(2,1,3) CONVOLUTIONAL CODE STATE MACHINE TABLE

u	S_n (m_1, m_2, m_3)	S_{n+1} (u, m_1, m_2)	V_1 $(u + m_2 + m_3)$	V_2 $(u + m_1 + m_2 + m_3)$
0	(0,0,0)	(0,0,0)	0	0
1	(0,0,0)	(1,0,0)	1	1
0	(0,0,1)	(0,0,0)	1	1
1	(0,0,1)	(1,0,0)	0	0
0	(0,1,0)	(0,0,1)	1	1
1	(0,1,0)	(1,0,1)	0	0
0	(0,1,1)	(0,0,1)	0	0
1	(0,1,1)	(1,0,1)	1	1
0	(1,0,0)	(0,1,0)	0	1
1	(1,0,0)	(1,1,0)	1	0
0	(1,0,1)	(0,1,0)	1	0
1	(1,0,1)	(1,1,0)	0	1
0	(1,1,0)	(0,1,1)	1	0
1	(1,1,0)	(1,1,1)	0	1
0	(1,1,1)	(0,1,1)	0	1
1	(1,1,1)	(1,1,1)	1	0

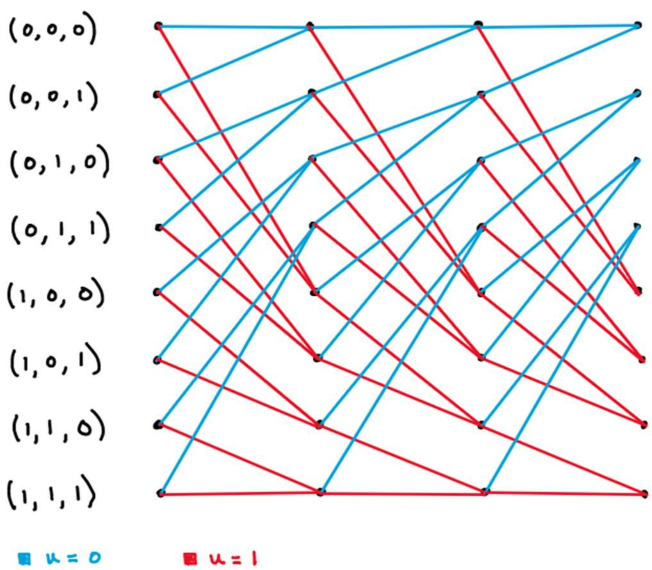

Fig. 7. (2,1,3) convolutional code trellis structure

III. RESULTS AND DISCUSSION

The simulation results for BER of BPSK modulation are provided in Fig. 8. The bit error rate decreases at a dramatic rate when SNR exceeds five decibels (dB). This is expected behavior when considering the effects of noise on a received signal. For larger values of SNR, the noise magnitude on average is so miniscule that, subsequently, there is also a small probability of a received signal crossing the decision boundary due to phase shifts. As SNR decreases, the BER curve starts to flatten out. The bit error rate reaches 1% for 4.202 < SNR < 4.437. When the SNR is 0 dB, the BER is approximately 7.88%.

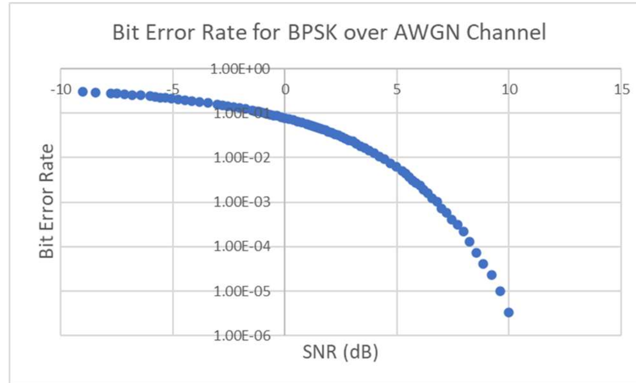

Fig. 8. BER versus SNR for BPSK modulation

The simulation results for error rates associated with QPSK modulation are provided in Fig. 9 and Fig. 10. The shape of the error curves for BER and SER are very similar implying a similar behavior with respect to SNR. For Fig. 9, SNR is provided in terms of energy per bit. For Fig. 10, SNR is provided in terms of energy per symbol. The BER for QPSK experiences a large drop off when SNR exceeds 5 dB. This same drop off behavior is present in the SER curve when SNR surpasses 8.5 dB. While the simulated BER curve of QPSK is almost the same as that of BPSK, it is important to note that this is due to the error rate being presented as a function of energy per bit. In the case where the symbol energy for both BPSK and QPSK is equivalent, the energy per bit for QPSK is halved. QPSK presents a larger SER than that of BPSK due to the larger susceptibility to phase ambiguity at the receiver. Consequently, it requires a higher SNR to achieve the same level of error rate performance as BPSK.

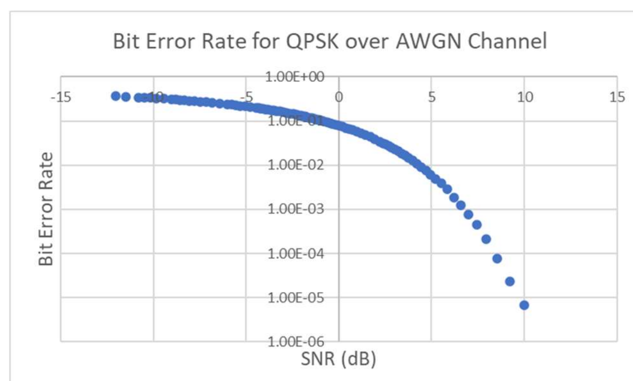

Fig. 9. BER versus SNR for QPSK modulation

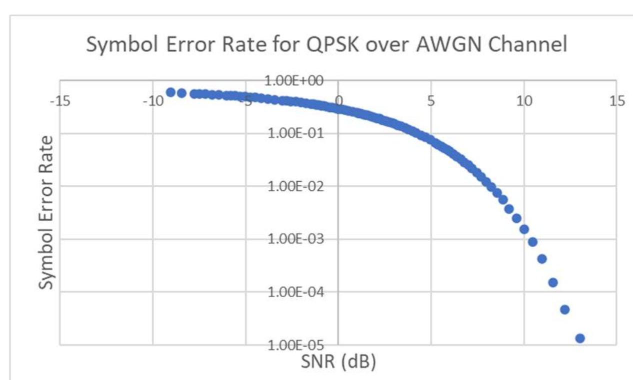

Fig. 10. SER versus SNR for QPSK modulation

The simulation results for error rates associated with 8-PSK modulation are provided in Fig. 11 and Fig. 12. Once again, the BER and SER curves behave similarly as a function of SNR. The BER for 8-PSK displays a drop in the detector performance as a function of bit energy when compared with BPSK and QPSK. The BER is affected both by the SER as well as the bit to symbol mapping. A large drop off in BER can be seen when SNR exceeds 9 dB. In addition, this same behavior presents itself for SER when SNR exceeds 12 dB. These values are greater than the ones described for BPSK and QPSK, further emphasizing the tradeoff between spectral efficiency and error rate.

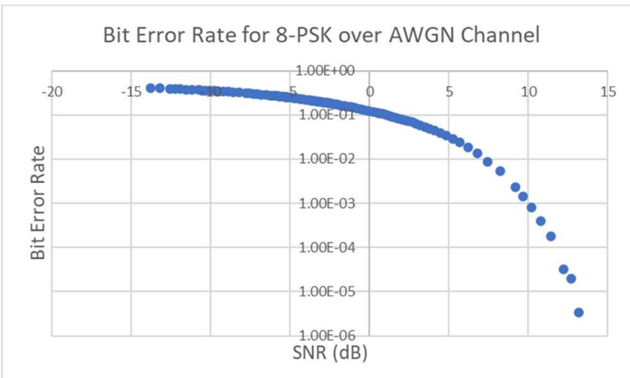

Fig. 11. BER versus SNR for 8-PSK modulation

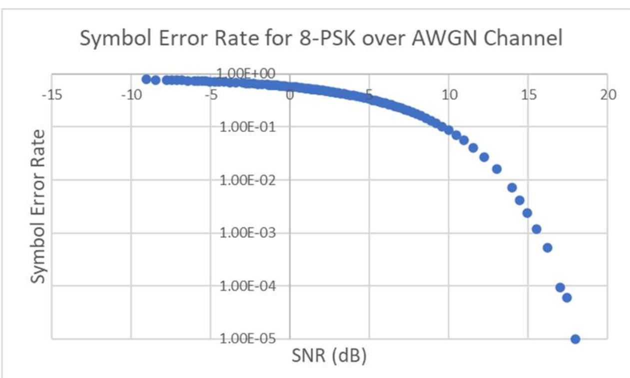

Fig. 12. SER versus SNR for 8-PSK modulation

To provide an error rate comparison for the different PSK modulation schemes that were simulated, the SNR for Fig. 13 and Fig. 14 are provided in terms of symbol energy. By using symbol energy, it is easier to compare the necessary SNR required for the transmitted signal to achieve the same error rate between BPSK, QPSK, and 8-PSK. When observing the plot for BER, as SNR rises, the difference between error rates on a relative scale increases significantly. It is also clear that as the number of symbols increases, the SNR at which the curve experiences a steep drop off for error rate also increases. A closer inspection of the 8-PSK BER curve reveals an inconsistent slope for larger values of SNR. This can be explained both by the sample size of the generated bit stream as well as the behavior of the detector. When an incorrect symbol is detected, the resulting number of bit errors could be one, two, or three bits, depending on the difference between the transmitted and received symbols. When looking at the SER plot, the symbol error rate looks to have a consistent slope for 8-PSK. This suggests that a smaller SER may contribute to a larger BER in situations where the incorrect number of decoded bits is also dependent on the incorrect symbol that was detected. It also highlights the importance of the mapping between bits and symbols due to the dependency of BER on the specific symbol that was incorrectly detected. The comparison of the different PSK modulation schemes provides valuable insight into the performance characteristics of these modulation techniques under different operating conditions.

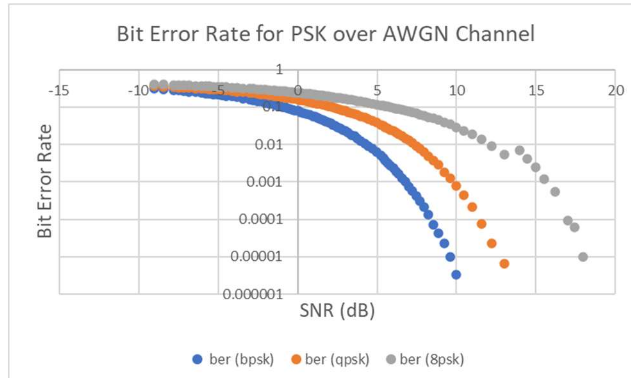

Fig. 13. BER versus SNR for PSK modulation

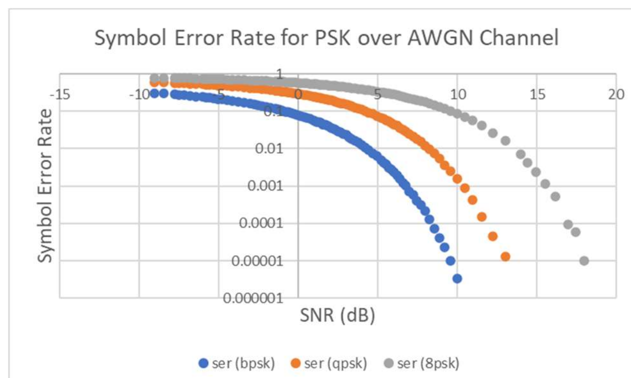

Fig. 14. SER versus SNR for PSK modulation

IV. Viterbi Algorithm for Error Correction

The results of the simulation involving convolutional encoding and the Viterbi algorithm for error correction illustrate that the BER performance of PSK modulation can be improved for sufficient levels of SNR. This is something that the designer of a communication system can implement to achieve better bit error rate performance at a cost of energy for redundancy in the code. The code that was used for this simulation encodes one input bit to two output bits resulting in twice the required power for transmission.

The results for Viterbi algorithm error correction on BPSK are provided in Fig. 15. The Viterbi algorithm does not achieve improvements until SNR exceeds approximately -1.761 dB. The relative BER improvements become more significant as SNR increases. For reference, when the SNR is 4 dB, the Viterbi algorithm achieves a BER that is approximately three hundred and eighty times better than the output from the optimal detector alone. These results highlight the potential of the Viterbi algorithm for improving the BER performance of BPSK modulation systems, particularly at higher SNRs.

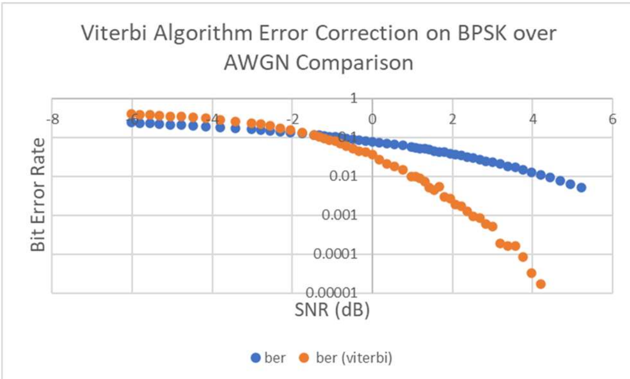

Fig. 15. Viterbi Algorithm BER comparison for BPSK

The results for Viterbi algorithm error correction on QPSK are provided in Fig. 16. The SNR in this plot is a function of the bit energy and therefore displays a close similarity to the BER for BPSK. The Viterbi algorithm does not achieve improvements until SNR exceeds approximately -1.584 dB, and relative BER improvements become more significant as SNR increases. To give another reference, when the SNR is 4 dB, the Viterbi algorithm achieves a BER that is approximately one hundred and nine times better than the output from the optimal detector alone. These results showcase the potential of the Viterbi algorithm for improving the BER of QPSK modulation systems, especially as SNR increases.

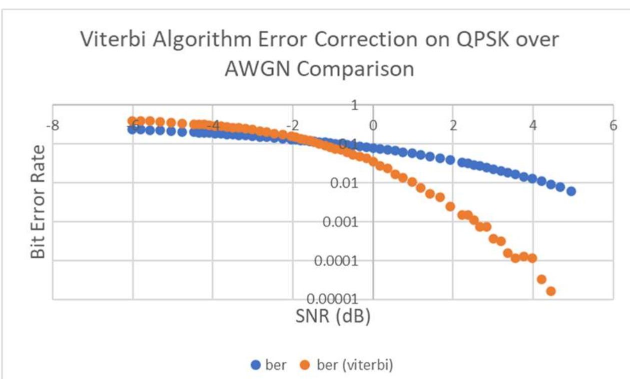

Fig. 16. Viterbi Algorithm BER comparison for QPSK

The results for Viterbi algorithm error correction on 8-PSK are provided in Fig. 17. The SNR in this plot is also a function of the bit energy. The Viterbi algorithm does not achieve improvements until SNR exceeds approximately 0 dB. Once again, the relative BER improvements become more significant as SNR increases. To provide a reference, when the SNR is 4 dB, the Viterbi algorithm achieves a BER that is approximately twelve and a half times better than the output from the optimal detector alone. As the SNR increases, the potential of the Viterbi algorithm for improving BER of 8-PSK modulation systems becomes more evident, as demonstrated by these results.

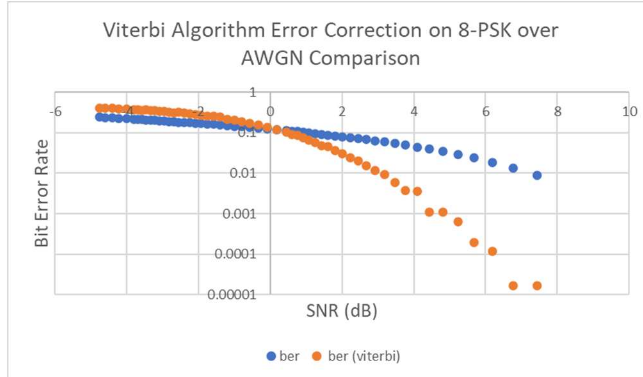

Fig. 17. Viterbi Algorithm BER comparison for 8-PSK

V. CONCLUSION

The simulated results for a communication system utilizing PSK modulation provide insight into BER and SER performance as a function of SNR. The data plots help to visualize the tradeoffs associated with spectral efficiency and error rate at the detector. In addition, the simulation investigated BER improvements that could be obtained by a communication system when implementing convolutional encoding and the Viterbi algorithm in addition to PSK modulation. The results for this part of the simulation show that for larger SNR, drastic BER improvements can be achieved.

REFERENCES

[1] A. Grami, "Passband Digital Transmission," Introduction to Digital Communications, pp. 299–355, 2016.
[2] K. Gilman, "Implementation of Viterbi Detection Algorithm for Convolutional Codes," Feb. 2023.